

**I Semester B.C.A. Examination, February/March 2024
(NEP Scheme) (Freshers and Repeaters)****MATHEMATICS – I
Mathematical Foundation**

Time : 2½ Hours

Max. Marks : 60

Instruction : Answer all the Sections.**SECTION – A**Answer **any six** of the following questions.**(6×2=12)**

1. Define proposition. Give an example.
2. Write truth table for $\sim p \vee \sim q$.
3. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 7, 8\}$ find $A \cup B$ and $B \cap A$.
4. Define union of two sets. Give an example.
5. If $A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}$ then find $2A + 3B$.
6. Define square matrix. Give an example.
7. If $G = \{7, 8\}$ and $H = \{5, 4, 2\}$ then find $G \times H$ and $H \times G$.
8. If $y = 3x^4 - 2x^3 + x + 8$ then find $\frac{dy}{dx}$.
9. Evaluate $\lim_{x \rightarrow 2} [x^2 + 2x - 3]$.

SECTION – BAnswer **any four** of the following questions.**(4×6=24)**

10. Prove that $(p \wedge q) \wedge \sim(p \vee q)$ is a contradiction.
11. If $A = \{1, 4\}$, $B = \{2, 3, 6\}$ and $C = \{2, 3, 7\}$ then verify $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
12. Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 2x$ is one-one and onto.

P.T.O.



13. Solve : $3x + 3y + 5z = 0$

$$3x + 5y + 9z = 0$$

$$5x + 9y + 17z = 0 \text{ by Cramer's rule.}$$

14. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and hence find A^{-1} .

15. If $y = 5x^6 + 6x^5 + 4x^4 + 3x^2 + 2x + 1$ then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 2$ and $x = 3$.

SECTION – C

Answer **any three** of the following questions.

(3×8=24)

16. a) Show that $(p \rightarrow q) \Leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.

b) Prove that $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

17. a) Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

b) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

i) $A \cup B$

ii) $A \cap B$

iii) $A - B$

iv) $B - A$.

18. a) If $A = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$, then Prove that $(A')' = A$.

b) If $A = \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$ then show that

$$A + (B + C) = (A + B) + C \text{ and find } 3B + 2C.$$

19. a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ by reducing to echelon form.

b) Find the eigen values for the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

20. a) Prove that the function $f(x) = 2x^3 - 21x^2 + 36x - 30$ is maximum at $x = 1$.

b) Find the minimum value of $f(x) = x^3 + 1$.